ON THE ERROR IN DETERMINING THE STRESS CONCENTRATION AT A FREE HOLE BY PLANE ELASTICITY THEORY METHODS

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The possibility of applying plane elasticity theory methods to compute the stress concentration at the surface of a free hole in a thick slab is studied herein. Asymptotic representations of the solution of the three-dimensional elasticity theory problem for a thick slab, obtained in [1], are utilized. It is shown that application of plane elasticity theory methods for a hole sufficiently remote from the outer contour of the slab is valid. The error obtained in the computations of the stress concentration by these methods is estimated.

1. Let us consider a homogeneous isotropic slab of thickness $2h$, bounded by cylindrical lateral surface Γ_1. Let there be a hole in the slab, which is bounded by a cylindrical surface Γ_2 sufficiently remote from the outer surface Γ_1 as compared with the slab thickness. Let a denote the major diameter of the hole. The surface Γ_1 is loaded by a system of stress resultants statically equivalent to zero and symmetric relative to the middle plane of the slab (the case of so-called slab compression is studied; the slab bending case has been studied in [2]). The surface of the hole Γ_2 is free of loading. The plane endfaces are also not loaded (Fig. 1). As is known, in this case stress concentrations will occur at Γ_2.

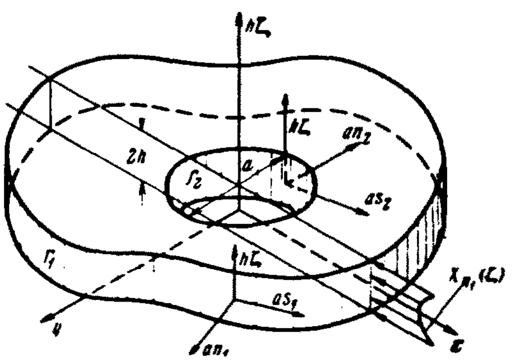

Fig. 1

Methods of computing the stress concentrations in such problems by using complex variable function theory and other plane elasticity theory methods are quite well developed and described in many publications, (see e.g. [3]).

It has been shown in [1] that additional members in the stresses on the loaded lateral surface of a thick slab will be quantities of the same order as for the solution of the plane problem, i.e. the plane elasticity theory solution on the loaded surface does not yield any true picture [4]. Let us examine what the situation is if the hole surface is stress-free.

2. As in [1], let us introduce nondimensional local coordinates n_1, s_1, ζ connected with the contour Γ_1, and n_2, s_2, ζ with Γ_2. The radius of curvature of the outer contour is $R_1(s_1)$, and of the inner contour $R_2(s_2)$. The direction cosines of the normal for the outer contour are $l_1(s_1)$, $m_1(s_1)$, and $l_2(s_2)$, $m_2(s_2)$ for the inner contour. A system of stress resultants $N_1(s_1, \zeta)$, $T_1(s_1, \zeta)$, $Z_1(s_1, \zeta)$ or $X_{n_1}(s_1, \zeta)$, $Y_{n_1}(s_1, \zeta)$, $Z_{n1}(s_1, \zeta)$ is given on the outer contour.

According to [1], the state of stress within a solid slab bounded by a surface Γ_1 loaded by such a system of stress resultants, is described by Formulas

$$\sigma_{n_1}{}^\circ = -\frac{1}{2}\left\{(m_1 + il_1)\frac{\partial}{\partial S_1}[\varphi^\circ + z\overline{(\varphi_0)'} + \overline{\psi^\circ}] + (m_1 - il_1)\frac{\partial}{\partial S_1}(\overline{\varphi^\circ} + \bar z(\varphi^\circ)' + \psi^\circ]\right\} -$$
$$-\lambda^2a^2\frac{\nu-1}{3\nu-1}\left(\frac{1}{3}-\zeta^2\right)\left[(m_1 + il_1)\frac{\partial}{\partial S_1}\overline{(\varphi^\circ)''} + (m_1 - il_1)\frac{\partial}{\partial S_1}(\varphi^\circ)''\right] -$$
$$-\frac{2\mu}{a}\sum_{k=1}^{\infty}\left(1-\frac{a}{2R_1}n_1+\frac{3a^2}{8R_1^2}n_1^2+\cdots\right)[(\nu-1)p_k(\zeta)+\delta_k{}^2\alpha_k(\zeta)]a_{k_1}{}^\circ(s_1)\exp\frac{\delta_k n_1}{\lambda} +$$
$$+\frac{2\mu}{a}\lambda\sum_{k=1}^{\infty}\left\{\left(1-\frac{a}{2R_1}n_1+\frac{3a^2}{8R_1^2}n_1^2+\cdots\right)[(\nu-1)p_k(\zeta)+\delta_k{}^2\alpha_k(\zeta)]a_{k2}{}^\circ(s_1)+\right.$$
$$+\left[-(\nu-1)\frac{n_1}{2\delta_k}p_k(\zeta)\left(\frac{a^2}{4R_1^2}+\frac{d^2}{ds_1^2}+\cdots\right)+\delta_k\alpha_k(\zeta)\left(-\frac{a}{R_1}+\frac{11a^2}{8R_1^2}n_1-\frac{n_1}{2}\frac{d^2}{ds_1^2}+\cdots\right)\right]\times$$
$$\left.\times a_{k1}{}^\circ(s_1)\right\}\exp\frac{\delta_k n_1}{\lambda}+\frac{2\mu}{a}2\nu\frac{1}{H_1}\lambda\sum_{p=1}^{\infty}\rho_p\cos(\rho_p\zeta)\frac{\partial}{\partial s_1}\left[\left(1-\frac{a}{2R_1}n_1+\frac{3a^2}{8R_1^2}n_1^2+\cdots\right)\times\right.$$
$$\left.\times f_{p1}^\circ(s_1)\right]\exp\frac{\rho_p n_1}{\lambda}+\cdots \tag{2.1}$$

$$\tau_{n_1s_1}^\circ = -\frac{1}{2}\left\{(l_1 - im_1)\frac{\partial}{\partial S_1}[\varphi^\circ + z\overline{(\varphi^\circ)'} + \overline{\psi^\circ}] + (l_1 + im_1)\frac{\partial}{\partial S_1}[\overline{\varphi^\circ} + \bar z(\varphi^\circ)' + \psi^\circ]\right\} -$$
$$-\lambda^2a^2\frac{\nu-1}{3\nu-1}\left(\frac{1}{3}-\zeta^2\right)\left[(l_1 - im_1)\frac{\partial}{\partial S_1}\overline{(\varphi^\circ)''} + (l_1 + im_1)\frac{\partial}{\partial S_1}(\varphi^\circ)''\right] -$$
$$-\frac{2\mu}{a}\nu\sum_{p=1}^{\infty}\rho_p\cos(\rho_p\zeta)\left(1-\frac{a}{2R_1}n_1+\frac{3a^2}{8R_1^2}n_1^2+\cdots\right)f_{p1}{}^\circ(s_1)\exp\frac{\rho_p n_1}{\lambda} +$$
$$-\frac{2\mu}{a}\lambda\left\{\frac{1}{H_1}\sum_{k=1}^{\infty}\delta_k\alpha_k(\zeta)\frac{\partial}{\partial s_1}\left[\left(1-\frac{a}{2R_1}n_1+\frac{3a^2}{8R_1^2}n_1^2+\cdots\right)a_{k1}{}^\circ(s_1)\right]\exp\frac{\delta_k n_1}{\lambda}+\right.$$
$$-\nu\sum_{p=1}^{\infty}\rho_p\cos(\rho_p\zeta)\left[\left(1-\frac{a}{2R_1}n_1+\frac{3a^2}{8R_1^2}n_1^2+\cdots\right)\left(\frac{a}{R_1}\frac{1}{H_1}f_{p1}{}^\circ-\rho_p f_{p2}{}^\circ\right)-\right.$$
$$\left.\left.-\left(-\frac{a}{R_1}+\frac{11a^2}{8R_1^2}n_1-\frac{n_1}{2}\frac{d^2}{ds_1^2}+\cdots\right)f_{p1}{}^\circ(s_1)\right]\exp\frac{\rho_p n_1}{\lambda}\right\}+\cdots \tag{2.2}$$

$$\tau_{n_1z}{}^\circ = \frac{2\mu}{a}\nu\sum_{k=1}^{\infty}\delta_k\gamma_k(\zeta)\left(1-\frac{a}{2R_1}n_1+\frac{3a^2}{8R_1^2}n_1^2+\cdots\right)a_{k1}{}^\circ(s_1)\exp\frac{\delta_k n_1}{\lambda}+$$
$$+\frac{2\mu}{a}\nu\lambda\left\{\sum_{k=1}^{\infty}\gamma_k(\zeta)\left[\delta_k\left(1-\frac{a}{2R_1}n_1+\frac{3a^2}{8R_1^2}n_1^2+\cdots\right)a_{k2}{}^\circ(s_1)+\right.\right.$$
$$\left.+\left(-\frac{a}{2R_1}+\frac{5a^2}{8R_1^2}n_1-\frac{n_1}{2}\frac{d^2}{ds_1^2}+\cdots\right)a_{k1}{}^\circ(s_1)\right]\exp\frac{\delta_k n_1}{\lambda}+$$
$$\left.+\frac{1}{H_1}\sum_{p=1}^{\infty}\rho_p\sin(\rho_p\zeta)\frac{\partial}{\partial s_1}\left[\left(1-\frac{a}{2R_1}n_1+\frac{3a^2}{8R_1^2}n_1^2+\cdots\right)f_{p1}{}^\circ(s_1)\right]\exp\frac{\rho_p n_1}{\lambda}\right\}+\cdots \tag{2.3}$$

$$\sigma_{s_1}{}^\circ = 2[(\varphi^\circ)' + \overline{(\varphi^\circ)'}] + \frac{1}{2}\left\{(m_1 + il_1)\frac{\partial}{\partial S_1}[\varphi^\circ + z\overline{(\varphi^\circ)'} + \overline{\psi^\circ}]+\right.$$
$$\left.+(m_1 - il_1)\frac{\partial}{\partial S_1}[\overline{\varphi^\circ} + \bar z(\varphi^\circ)' + \psi^\circ]\right\}+$$

$$+\lambda^2 a^2 \frac{\nu-1}{3\nu-1}\left(\frac{1}{3}-\zeta^2\right)\left[(m_1+il_1)\frac{\partial}{\partial S_1}\overline{(\varphi^\circ)''}+(m_1-il_1)\frac{\partial}{\partial S_1}(\varphi^\circ)''\right]+$$

$$+\frac{2\mu}{a}(\nu-1)\sum_{k=1}^{\infty}p_k(\zeta)\left(1-\frac{a}{2R_1}n_1+\frac{3a^2}{8R_1^2}n_1^2+\cdots\right)a_{k1}^\circ(s_1)\exp\frac{\delta_k n_1}{\lambda}+$$

$$+\frac{2\mu}{a}\lambda\sum_{k=1}^{\infty}\left\{\left(1-\frac{a}{2R_1}n_1+\frac{3a^2}{8R_1^2}n_1^2+\cdots\right)\left[(\nu-1)p_k(\zeta)a_{k2}^\circ(s_1)+\frac{1}{H_1}\frac{a}{R_1}\delta_k a_{k1}^\circ(s_1)\right]-\right.$$

$$\left.-(\nu-1)n_1\frac{p_k(\zeta)}{2\delta_k}\left(\frac{a^2}{4R_1^2}+\frac{d^2}{ds_1^2}+\cdots\right)a_{k1}^\circ(s_1)\right\}\exp\frac{\delta_k n_1}{\lambda}- \tag{2.4}$$

$$-\frac{2\mu}{a}2\nu\frac{1}{H_1}\lambda\sum_{p=1}^{\infty}\rho_p\cos(\rho_p\zeta)\frac{\partial}{\partial s_1}\left[\left(1-\frac{a}{2R_1}n_1+\frac{3a^2}{8R_1^2}n_1^2+\cdots\right)f_{p1}^\circ(s_1)\right]\exp\frac{\rho_p n_1}{\lambda}+\cdots$$

$$\tau_{s_1 z}^\circ=\frac{2\mu}{a}\nu\sum_{p=1}^{\infty}\rho_p^2\sin(\rho_p\zeta)\left(1-\frac{a}{2R_1}n_1+\frac{3a^2}{8R_1^2}n_1^2+\cdots\right)f_{p1}^\circ(s_1)\exp\frac{\rho_p n_1}{\lambda}+$$

$$+\frac{2\mu}{a}\nu\lambda\left\{\frac{1}{H_1}\sum_{k=1}^{\infty}\gamma_k(\zeta)\frac{\partial}{\partial s_1}\left[\left(1-\frac{a}{2R_1}n_1+\frac{3a^2}{8R_1^2}n_1^2+\cdots\right)a_{k1}^\circ(s_1)\right]\exp\frac{\delta_k n_1}{\lambda}+\right.$$

$$+\sum_{p=1}^{\infty}\rho_p\sin(\rho_p\zeta)\left[\rho_p\left(1-\frac{a}{2R_1}n_1+\frac{3a^2}{8R_1^2}n_1^2+\cdots\right)f_{p2}^\circ(s_1)\right]+\left[\left(-\frac{a}{2R_1}+\frac{5a^2}{8R_1^2}n_1-\right.\right.$$

$$\left.\left.\left.-\frac{n_1}{2}\frac{d^2}{ds_1^2}+\cdots\right)f_{p1}^\circ(s_1)\right]\exp\frac{\rho_n n_1}{\lambda}\right\}+\cdots \tag{2.5}$$

$$\sigma_z^\circ=\frac{2\mu}{a}\nu\sum_{K=1}^{\infty}q_k(\zeta)\left\{\left(1-\frac{a}{2R_1}n_1+\frac{3a^2}{8R_1^2}n_1^2+\cdots\right)a_{k1}^\circ(s_1)+\right.$$

$$+\lambda\left[\left(1-\frac{a}{2R_1}n_1+\frac{3a^2}{8R_1^2}n_1^2+\cdots\right)a_{K2}^\circ(s_1)\right]-$$

$$\left.-\frac{n_1}{2\delta_k}\left(\frac{a^2}{4R_1^2}+\frac{d^2}{ds_1^2}+\cdots\right)a_{K1}^\circ(s_1)\right]+\cdots\right\}\exp\frac{\delta_k n_1}{\lambda} \tag{2.6}$$

As in [1], here λ denotes the nondimensional slab-thickness parameter $\lambda=h/a$; i is the imaginary unit, and the bar over the symbol denotes the somplex conjugate. All the remaining notation has been explained in [1].

The functions of a complex variable $\varphi^\circ(z)$ and $\psi^\circ(z)$ which are analytic within Γ_1, are representable by the series

$$\varphi^\circ(z)=\varphi_0^\circ(z)+\lambda\varphi_1^\circ(z)+\lambda^2\varphi_2^\circ(z)+\ldots \tag{2.7}$$

$$\psi^\circ(z)=\psi_0^\circ(z)+\lambda\psi_1^\circ(z)+\lambda^2\psi_2^\circ(z)+\ldots \tag{2.8}$$

where $\varphi_0^\circ(z)$ and $\psi_0^\circ(z)$ are functions whose boundary values are determined by the Kolosov-Muskhelishvili conditions for this problem [5]. The contour values for the remaining functions $\varphi_i^\circ(z)$ and $\psi_i^\circ(z)$, as well as for the functions $a_{ki}^\circ(s_1)$ and $f_{pi}^\circ(s_1)$ are determined by the boundary conditions on Γ_1. The functions a°_{ki} are found from some infinite system of linear algebraic equations.

It is seen from (2.1)–(2.6) that on leaving Γ_1 for the depth of the slab ($n_1\to-\infty$) the state of stress described by the functions $a_k^\circ(s_1$ and $f_p^\circ(s_1)$ damp exponentially and only the biharmonic state of stress is propagated deep into slab. It can be assumed that the state of stress of the slab at a sufficient distance from the outer contour Γ_1 will be determined by Formulas [6]

$$\sigma_x{}^\circ = \frac{1}{2}[2(\varphi^\circ)' + 2\overline{(\varphi^\circ)'} - \bar{z}(\varphi^\circ)'' - z\overline{(\varphi^\circ)''} - (\psi^\circ)' - \overline{(\psi^\circ)'}] - $$
$$- \lambda^2 a^2 \frac{\nu-1}{3\nu-1}\left(\frac{1}{3} - \zeta^2\right)[(\varphi^\circ)''' + \overline{(\varphi^\circ)'''}]$$
$$\sigma_y{}^\circ = \frac{1}{2}[2(\varphi^\circ)' + 2\overline{(\varphi^\circ)'} + \bar{z}(\varphi^\circ)'' + z\overline{(\varphi^\circ)''} + (\psi^\circ)' + \overline{(\psi^\circ)'}] +$$
$$+ \lambda^2 a^2 \frac{\nu-1}{3\nu-1}\left(\frac{1}{3} - \zeta^2\right)[(\varphi^\circ)''' + \overline{(\varphi^\circ)'''}]$$
$$\tau_{xy}{}^\circ = -\frac{1}{2} i [\bar{z}(\varphi^\circ)'' - z\overline{(\varphi^\circ)''} + (\psi^\circ)' - \overline{(\psi^\circ)'}] - \lambda^2 a^2 \frac{\nu-1}{3\nu-1}\left(\frac{1}{3} - \zeta^2\right) i [(\varphi^\circ)''' - \overline{(\varphi^\circ)'''}]$$
$$\tau_{xz}{}^\circ = \tau_{yz}{}^\circ = \sigma_z{}^\circ = 0 \tag{2.9}$$

Since a solid slab without a hole is now considered, then the stresses

$$\sigma_{n_2}{}^\circ = -\frac{1}{2}\left\{(m_2 + il_2)\frac{d}{dS_2}[\varphi^\circ + z\overline{(\varphi^\circ)'} + \bar{\psi}^\circ] + (m_2 - il_2)\frac{d}{dS_2}[\bar{\varphi}^\circ + \bar{z}(\varphi^\circ)' + \psi^\circ]\right\} -$$
$$- \lambda^2 a^2 \frac{\nu-1}{3\nu-1}\left(\frac{1}{3} - \zeta^2\right)\left[(m_2 + il_2)\frac{d}{dS_2}\overline{(\varphi^\circ)''} + (m_2 - il_2)\frac{d}{dS_2}(\varphi)''\right] \tag{2.10}$$

$$\tau^\circ_{n_2 s_2} = -\frac{1}{2}\left\{(l_2 - im_2)\frac{d}{dS_2}[\varphi^\circ + z\overline{(\varphi^\circ)} + \bar{\psi}^\circ] + (l_2 + im_2)\frac{d}{dS_2}[\bar{\varphi}^\circ + \bar{z}(\varphi^\circ)' + \psi^\circ]\right\} -$$
$$- \lambda^2 a^2 \frac{\nu-1}{3\nu-1}\left(\frac{1}{3} - \zeta^2\right)\left[(l_2 - im_2)\frac{d}{dS_2}\overline{(\varphi^\circ)''} + (l_2 + im_2)\frac{d}{dS_2}(\varphi^\circ)''\right] \tag{2.11}$$

$$\sigma_{s_2}{}^\circ = 2[(\varphi^\circ)' + \overline{(\varphi^\circ)'}] + \frac{1}{2}\left\{(m_2 + il_2)\frac{d}{dS_2}[\varphi^\circ + z\overline{(\varphi^\circ)'} + \bar{\psi}^\circ] + (m_2 - il_2)\frac{d}{dS_2}[\bar{\varphi}^\circ + \right.$$
$$\left. + \bar{z}(\varphi^\circ)' + \psi^\circ]\right\} + \lambda^2 a^2 \frac{\nu-1}{3\nu-1}\left(\frac{1}{3} - \zeta^2\right)\left[(m_2 + il_2)\frac{d}{dS_2}\overline{(\varphi^\circ)''} + (m_2 - il_2)\frac{d}{dS_2}(\varphi^\circ)''\right] \tag{2.12}$$

$$\tau_{n_2 z}{}^\circ = \tau_{s_2 z}{}^\circ = \sigma_z{}^\circ = 0 \tag{2.13}$$

or the stresses

$$X_{n_2}{}^\circ + iY_{n_2}{}^\circ = -i\frac{d}{dS_2}[\varphi^\circ + z\overline{(\varphi^\circ)'} + \bar{\psi}^\circ] - 2\lambda^2 a^2 \frac{\nu-1}{3\nu-1}\left(\frac{1}{3} - \zeta^2\right) i \frac{d}{dS_2}\overline{(\varphi^\circ)''}, \quad Z_{n_2}{}^\circ = 0 \tag{2.14}$$

act on the surface Γ_1.

But there is actually a hole in the slab, bouded by the surface Γ_2, where this surface is free of loading. Therefore, Γ_2 must be freed of the stresses $\sigma_{n_2}{}^\circ$, $\tau^\circ_{n_2 s_2}$ which originate there because of the loading on the outer surface Γ_1. To do this, it is necessary to superpose a new state of stress corresponding to the solution of the elasticity theory problem for an infinite slab with a hole, on the biharmonic state of stress (2.9) acting within the slab. It should not alter the stresses on the surface Γ_1, but the values $-\sigma_{n_2}{}^\circ$, $-\tau^\circ_{n_2 s_2}$ should be taken on Γ_2, and it is understood that the endfaces should remain stress-free. We call this the reflected state of stress.

3. Let us construct the reflected state of stress on the basis of the method expounded in [1]. Since the exterior problem is hence solved, the potential and vortex states of stress should damp out with withdrawal from the edge into the depths of the slab, i.e. as $n_2 \to \infty$. It is easy to prove that the solution of the exterior problem can be obtained from the solution of the interior problem constructed in [1], by replacing δ_k by $-\delta_k$ and ρ_p by $-\rho_p$. Then the reflected state of stress will be

$$\sigma_{n_2}{}^* = -\frac{1}{2}\left\{(m_2 + il_2)\frac{\partial}{\partial S_2}[\varphi^* + z\overline{(\varphi^*)'} + \bar{\psi}^*] + (m_2 - il_2)\frac{\partial}{\partial S_2}[\bar{\varphi}^* + \bar{z}(\varphi^*)' + \psi^*]\right\} -$$

$$- \lambda^2 a^2 \frac{\nu-1}{3\nu-1}\left(\frac{1}{3}-\zeta^2\right)\left[(m_2+il_2)\frac{\partial}{\partial S_2}\overline{(\varphi^*)''} + (m_2-il_2)\frac{\partial}{\partial S_2}(\varphi^*)''\right] +$$

$$+\frac{2\mu}{a}\sum_{k=1}^{\infty}\left(1-\frac{a}{2R_2}n_2+\frac{3a^2}{8R_2^2}n_2^2+\cdots\right)[(\nu-1)p_k(\zeta)+\delta_k^2\alpha(\zeta)]\,a_{k1}^*(s_2)\exp\left(-\frac{\delta_k n_2}{\lambda}\right)+$$

$$+\frac{2\mu}{a}\lambda\sum_{k=1}^{\infty}\left\{\left(1-\frac{a}{2R_2}n_2+\frac{3a^2}{8R_2^2}n_2^2+\cdots\right)[(\nu-1)p_k(\zeta)+\delta_k^2\alpha_k(\zeta)]\,a_{k2}^*(s_2)+\right.$$

$$+\left[(\nu-1)\frac{n_2}{2\delta_K}p_k(\zeta)\left(\frac{a^2}{4R_2^2}+\frac{d^2}{ds_2^2}+\cdots\right)-\delta_k\alpha_k(\zeta)\left(-\frac{a}{H_2}+\frac{11a^2}{8R_2^2}n_2-\frac{n_2}{2}\frac{d^2}{ds_2^2}+\cdots\right)\right]\times$$

$$\left.\times\, a_{k1}^*(s_2)\right\}\exp\left(-\frac{\delta_k n_2}{\lambda}\right)-\frac{2\mu}{a}2\nu\frac{1}{H_2}\lambda\sum_{p=1}^{\infty}\rho_p\cos(\rho_p\zeta)\frac{\partial}{\partial s_2}\left[\left(1-\frac{a}{2R_2}n_2+\right.\right.$$

$$\left.\left.+\frac{3a^2}{8R_2^2}n_2^2+\cdots\right)f_{p1}^*(s_2)\right]\exp\left(-\frac{\rho_p n_2}{\lambda}\right)+\cdots \qquad (3.1)$$

$$\tau_{n_2s_2}^* = -\frac{1}{2}\left\{(l_2-im_2)\frac{\partial}{\partial S_2}[\varphi^*+z\overline{(\varphi^*)'}+\overline{\psi^*}]+(l_2+im_2)\frac{\partial}{\partial S_2}[\overline{\varphi^*}+\bar z(\varphi^*)'+\psi^*]\right\}-$$

$$-\lambda^2a^2\frac{\nu-1}{3\nu-1}\left(\frac{1}{3}-\zeta^2\right)\left[(l_2-im_2)\frac{\partial}{\partial S_2}\overline{(\varphi^*)''}+(l_2+im_2)\frac{\partial}{\partial S_2}(\varphi^*)''\right]-$$

$$-\frac{2\mu}{a}\nu\sum_{p=1}^{\infty}\rho_p^2\cos(\rho_p\zeta)\left(1-\frac{a}{2R_2}n_2+\frac{3a^2}{8R_2^2}n_2^2+\cdots\right)f_{p1}^*(s_2)\exp\left(-\frac{\rho_p n_2}{\lambda}\right)-$$

$$-\frac{2\mu}{a}\lambda\left\{\frac{1}{H_2}\sum_{k=1}^{\infty}\delta_k\alpha_k(\zeta)\frac{\partial}{\partial S_2}\left[\left(1-\frac{a}{2R_2}n_2+\frac{3a^2}{8R_2^2}n_2^2+\cdots\right)a_{k1}^*(s_2)\right]\exp\left(-\frac{\delta_k n_2}{\lambda}\right)+\right.$$

$$+\nu\sum_{p=1}^{\infty}\rho_p\cos(\rho_p\zeta)\left[\left(1-\frac{a}{2R_2}n_2+\frac{3a^2}{8R_2^2}n_2^2+\cdots\right)\left(\frac{a}{R_2}\frac{1}{H_2}f_{p1}^*+\rho_pf_{p2}^*\right)-\right.$$

$$\left.\left.-\left(-\frac{a}{R_2}+\frac{11a_2}{8R_2^2}n_2-\frac{n_2}{2}\frac{d^2}{ds_2^2}+\cdots\right)f_{p1}^*(s_2)\right]\exp\left(-\frac{\rho_p n_2}{\lambda}\right)\right\}+\cdots \qquad (3.2)$$

$$\tau_{n_2z}^* = -\frac{2\mu}{a}\nu\sum_{k=1}^{\infty}\delta_k\gamma_k(\zeta)\left(1-\frac{a}{2R_2}n_2+\frac{3a^2}{8R_2^2}n_2^2+\cdots\right)a_{k1}^*(s_2)\exp\left(-\frac{\delta_k n_2}{\lambda}\right)+$$

$$+\frac{2\mu}{a}\nu\lambda\left\{\sum_{k=1}^{\infty}\gamma_k(\zeta)\left[-\delta_k\left(1-\frac{a}{2R_2}n_2+\frac{3a^2}{8R_2^2}n_2^2+\cdots\right)a_{k2}^*(s_2)+\right.\right.$$

$$\left.+\left(-\frac{a}{2R_2}+\frac{5a^2}{8R_2^2}n_2-\frac{n_2}{2}\frac{d^2}{ds_2^2}+\cdots\right)a_{k1}^*(s_2)\right]\exp\left(-\frac{\delta_k n_2}{\lambda}\right)+$$

$$\left.+\frac{1}{H_2}\sum_{p=1}^{\infty}\rho_p\sin(\rho_p\zeta)\frac{\partial}{\partial s_2}\left[\left(1-\frac{a}{2R_2}n_2+\frac{3a^2}{8R_2^2}n_2^2+\cdots\right)f_{p1}^*(s_2)\right]\exp\left(-\frac{\rho_p n_2}{\lambda}\right)\right\}+\cdots \qquad (3.3)$$

$$\sigma_{s_2}^* = 2[(\varphi^*)'+\overline{(\varphi^*)'}]+\frac{1}{2}\left\{(m_2+il_2)\frac{\partial}{dS_2}[\varphi^*+z\overline{(\varphi^*)'}+\overline{\psi^*}]+\right.$$

$$\left.+(m_2-il_2)\frac{\partial}{\partial S_2}[\overline{\varphi^*}+\bar z(\varphi^*)'+\psi^*]\right\}+\lambda^2a^2\frac{\nu-1}{3\nu-1}\left(\frac{1}{3}-\zeta^2\right)\left[(m_2+il_2)\frac{\partial}{\partial S_2}\overline{(\varphi^*)''}+\right.$$

$$\left.+(m_2-il_2)\frac{\partial}{\partial S_2}(\varphi^*)''\right]+\frac{2\mu}{a}(\nu-1)\sum_{k=1}^{\infty}p_k(\zeta)\left(1-\frac{a}{2R_2}n_2+\frac{3a^2}{8R_2^2}n_2^2+\cdots\right)a_{k1}^*(s_2)\times$$

$$\times \exp\left(-\frac{\delta_k n_2}{\lambda}\right) + \frac{2\mu}{a}\lambda \sum_{k=1}^{\infty}\left\{\left(1-\frac{a}{2R_2}n_2+\frac{3a^2}{8R_2^2}n_2^2+\cdots\right)\left[(\nu-1)p_k(\zeta)a_{k2}^*(s_2)-\right.\right.$$

$$\left.-\frac{1}{H_2}\frac{a}{R_2}\delta_k a_{k1}^*(s_2)\right] + (\nu-1)n_2\frac{p_k(\zeta)}{2\delta_k}\left(\frac{a^2}{4R_2^2}+\frac{d^2}{ds_2^2}+\cdots\right)a_{k1}^*(s_2)\Big\}\exp\left(-\frac{\delta_k n_2}{\lambda}\right) + \quad (3.4)$$

$$+\frac{2\mu}{a}2\nu\frac{1}{H_2}\lambda\sum_{p=1}^{\infty}\rho_p\cos(\rho_p\zeta)\frac{\partial}{\partial s_2}\left[\left(1-\frac{a}{2R_2}n_2+\frac{3a^2}{8R_2^2}n_2^2+\cdots\right)f_{p1}^*(s_2)\right]\exp\left(-\frac{\rho_p n_2}{\lambda}\right)+\cdots$$

$$\tau_{s_2 z}^* = -\frac{2\mu}{a}\nu\sum_{p=1}^{\infty}\rho_p^2\sin(\rho_p\zeta)\left(1-\frac{a}{2R_2}n_2+\frac{3a^2}{8R_2^2}n_2^2+\cdots\right)f_{p1}^*(s_2)\exp\left(-\frac{\rho_p n_2}{\lambda}\right)+$$

$$+\frac{2\mu}{a}\nu\lambda\left\{\frac{1}{H_2}\sum_{k=1}^{\infty}\gamma_k(\zeta)\frac{\partial}{\partial s_2}\left[\left(1-\frac{a}{2R_2}n_2+\frac{3a^2}{8R_2^2}n_2^2+\cdots\right)a_{k1}^*(s_2)\exp\left(-\frac{\delta_k n_2}{\lambda}\right)+\right.\right.$$

$$+\sum_{p=1}^{\infty}\rho_p\sin(\rho_p\zeta)\left[-\rho_p\left(1-\frac{a}{2R_2}n_2+\frac{3a^2}{8R_2^2}n_2^2+\cdots\right)f_{p2}^*(s_2)+\right.$$

$$\left.\left.+\left(-\frac{a}{2R_2}+\frac{5a^2}{8R_2^2}n_2-\frac{n^2}{2}\frac{d^2}{ds_2^2}+\cdots\right)f_{p1}^*(s_2)\right]\exp\left(-\frac{\rho_p n_2}{\lambda}\right)\right\}+\cdots \quad (3.5)$$

$$\sigma_z^* = \frac{2\mu}{a}\nu\sum_{k=1}^{\infty}q_k(\zeta)\left\{\left(1-\frac{a}{2R_2}n_2+\frac{3a^2}{8R_2^2}n_2^2+\cdots\right)a_{k1}^*(s_2)+\lambda\left[\left(1-\frac{a}{2R_2}n_2+\right.\right.\right.$$

$$\left.\left.\left.+\frac{3a^2}{8R_2^2}n_2^2+\cdots\right)a_{k2}^*(s_2)+\frac{n_2}{2\delta_k}\left(\frac{a^2}{4R_2^2}+\frac{d^2}{ds_2^2}+\cdots\right)a_{k1}^*(s_2)\right]+\cdots\right\}\exp\left(-\frac{\delta_k n_2}{\lambda}\right) \quad (3.6)$$

Here $\varphi^*(z)$, $\psi^*(z)$ are functions of a complex variable, analytic outside the contour Γ_2 and which do not alter the stresses at infinity. They are represented by the series

$$\varphi^*(z) = \varphi_0^*(z) + \lambda\varphi_1^*(z) + \lambda^2\varphi_2^*(z) + \ldots \quad (3.7)$$

$$\psi^*(z) = \psi_0^*(z) + \lambda_1\psi_1^*(z) + \lambda^2\psi_2^*(z) + \ldots \quad (3.8)$$

4. The contour values for the functions $\varphi_i^*(z)$ and $\psi_i^*(z)$ and the functions $a_{ki}^*(s_2)$ and $f_{pi}^*(s_2)$ are determined from the boundary conditions on Γ_2 which have the form

$$N_2(s_2,\zeta) = -\sigma_{n_2}^\circ, \qquad T_2(s_2,\zeta) = -\tau_{n_2 s_2}^\circ, \qquad Z_2(s_2,\zeta) = 0 \quad (4.1)$$

or

$$X_{n_2}(s_2,\zeta) + iY_{n_2}(s_2,\zeta) = -(X_{n_2}^\circ + iY_{n_2}^\circ) \quad (4.2)$$

According to [1], the boundary conditions for the functions φ_0^*, ψ_0^* will be

$$\frac{d}{dS_2}[\varphi_0^* + z\overline{(\varphi_0^*)'} + \overline{\psi_0^*}] = i(\langle X_{n_20}\rangle + i\langle Y_{n_20}\rangle) \quad (4.3)$$

where the angular brackets denote mean values with respect to the height.

Taking account of boundary condition (4.2) and equality (2.14), we obtain the following boundary condition for φ_0^* and ψ_0^*

$$\frac{d}{dS_2}[\varphi_0^* + z\overline{(\varphi_0^*)'} + \overline{\psi_0^*}] = -\frac{d}{dS_2}[\varphi_0^\circ + z\overline{(\varphi_0^\circ)'} + \overline{\psi_0^\circ}] \quad (4.4)$$

It is hence seen that the state of stress described by the functions φ_0^* and ψ_0^* is a plane elasticity theory solution which removes stresses produced by the functions φ_0° and ψ_0°, from the surface of the hole Γ_2. This state of stress evidently does not change the stresses at infinity since φ^* and ψ^* are analytic functions outside Γ_2 and yield stresses which vanish at infinity. We call it the plane reflected state.

We have the following system of equations to determine the function $a_{k_1}^*(s_2)$

$$- 4\nu \sum_{\substack{k=1 \\ k \neq m}}^{\infty} \frac{\delta_k^2 \delta_m^2 (\sin^2\delta_m - \sin^2 \delta_k)}{(\delta_m^2 - \delta_k^2)(\delta_m - \delta_k)} [(\nu - 1)(\delta_k^2 + \delta_m^2) + 2(\nu + 1)\delta_k \delta_m] a_{k1}^* (s_2) +$$

$$+ 2\nu^2 \delta_m^3 \left(\frac{2}{3} \sin^2 \delta_m - 1\right) a_{m1}^* (s_2) = - \frac{a}{2\mu} \delta_m N_{m0} + \frac{a}{2\mu}(\nu - 1) \frac{\sin^2 \delta_m}{\delta_m} \times$$

$$\times \left\{(m_2 + il_2) \frac{d}{dS_2} [\varphi_0^* + z\overline{(\varphi_0^*)'} + \overline{\psi_0^*}] + (m_2 - il_2) \frac{d}{dS_2} [\overline{\varphi_0^*} + \bar{z}(\varphi_0^*)' + \psi_0^*]\right\}$$

$$(m = 1, 2, \ldots) \tag{4.5}$$

Here

$$N_{m0} = \int_{-1}^{1} N_{20}(s_2, \zeta)\, a_m(\zeta)\, d\zeta = -(\nu - 1) \frac{\sin^2 \delta_m}{\delta_m^2} \times$$

$$\times \left\{(m_2 + il_2) \frac{d}{dS_2} [\varphi_0^\circ + z\overline{(\varphi_0^\circ)'} + \overline{\psi_0^\circ}] + (m_2 - il_2) \frac{d}{dS_2} [\overline{\varphi_0^\circ} + \bar{z}(\varphi_0^\circ)' + \psi_0^\circ]\right\} \tag{4.6}$$

On the basis of boundary condition (4.4) it is seen that the right sides of the system (4.5) vanish. The system (4.5) is such that all the $a_{k_1}^* (s_2) \equiv 0$ here.

To determine $f_{f_1}^* (s_2)$ we obtain the equation

$$f_{t1}^* (s_2) = - \frac{a}{2\mu\nu\rho_t^2} T_{t0}(s_2) \qquad \begin{pmatrix} t = 1, 2, \ldots \\ \rho_t = t\pi \end{pmatrix} \tag{4.7}$$

$$T_{t0} = \int_{-1}^{1} T_{20}(s_2, \zeta) \cos(\rho_t \zeta)\, d\zeta = 0 \tag{4.8}$$

It hence follows at once that all the $f_{t_1}^* (s_2) \equiv 0$.

In the next approximation for the functions φ_1^* and ψ_1^* we obtain the boundary condition

$$\frac{d}{dS_2} [\varphi_1^* + z\overline{(\varphi_1^*)'} + \overline{\psi_1^*}] = - \frac{d}{dS_2} [\varphi_1^\circ + z\overline{(\varphi_0^\circ)'} + \overline{\psi_1^\circ}] \tag{4.9}$$

Let us note that although this boundary condition has the same form as (4.4), the functions φ_1^* and ψ_1^* will not be the solution of the plane elasticity theory problem since the boundary values for the functions φ_1° and ψ_1° do not agree with the Kolosov-Muskhelishvili conditions, but depend on the functions $a_{k_1}^\circ$, $f_{p_1}^\circ$ on the outer contour Γ_1.

For the functions a_{k2}^*, f_{p2}^* we find

$$\{a_{m2}^*\} = - \frac{a}{2\mu}\left(\delta_m N_{m1} + \frac{a}{2R_2} N_{m0} + \frac{dT_{m0}}{ds_2} - Z_{m1}\right) -$$

$$- \frac{a}{2\mu}(\nu - 1) \frac{\sin^2 \delta_m}{\delta_m^2} \left\{-\delta_m \left[(m_2 + il_2) \frac{d}{dS_2} (\varphi_1^* + z\overline{(\varphi_1^*)'} + \overline{\psi_1^*}) + (m_2 - il_2) \times\right.\right.$$

$$\left. \times \frac{d}{dS_2} (\overline{\varphi_1^*} + \bar{z}(\varphi_1^*)' + \psi_1^*)\right] + \frac{a}{2R_2} \left[(m_2 + il_2) \frac{d}{dS_2} (\varphi_0^* + z\overline{(\varphi_0^*)'} + \overline{\psi_0^*}) +\right.$$

$$\left. + (m_2 - il_2) \frac{d}{dS_2} (\overline{\varphi_0^*} + \bar{z}(\varphi_0^*)' + \psi_0^*)\right] - \left[(l_2 - im_2) \frac{d^2}{dS_2^2} (\varphi_0^* + z\overline{(\varphi_0^*)'} + \overline{\psi_0^*}) +\right.$$

$$\left.\left. + (l_2 + im_2) \frac{d^2}{dS_2^2} (\overline{\varphi_0^*} + \bar{z}(\varphi_0^*)' + \psi_0^*)\right]\right\}, \qquad m = 1, 2, \ldots \tag{4.10}$$

$$f_{t2}^* (s_2) = - \frac{a}{2\mu\rho_t^2}\left(\frac{dN_{t0}}{ds_2} + \rho_t T_{t1} + \frac{a}{2R_2} T_{t0}\right), \qquad t = 1, 2, \ldots \tag{4.11}$$

Here the braces { } denote the matrix of the system (4.5), which is constant in all the approximations

$$T_{t1} = \int_{-1}^{1} (-\tau_{n_2 s_2}{}^\circ)_1 \cos(\rho_t \zeta)\, d\zeta = 0, \quad Z_{m1} = \int_{-1}^{1} (-\tau_{n_2 z}{}^\circ)_1 \beta_m(\zeta)\, d\zeta = 0$$

$$N_{t0} = \int_{-1}^{1} (-\sigma_{n_2}{}^\circ)_0 \cos(\rho_t \zeta)\, d\zeta = 0 \tag{4.12}$$

$$N_{m1} = \int_{-1}^{1} (-\sigma_{n_2}{}^\circ)_1 \alpha_m(\zeta)\, d\zeta = -(\nu - 1) \frac{\sin^2 \delta_m}{\delta_m{}^2} \Big[(m_2 + il_2) \frac{d}{dS_2} (\varphi_1{}^\circ + z\, \overline{(\varphi_1{}^\circ)'} + \overline{\psi_1{}^\circ}) +$$
$$+ (m_2 - il_2) \frac{d}{dS_2} (\overline{\varphi_1{}^\circ} + \bar z (\varphi_1{}^\circ)' + \psi_1{}^\circ) \Big] \tag{4.13}$$

$$T_{m0} = \int_{-1}^{1} (-\tau_{n_2 s_2}{}^\circ)_0 \alpha_m(\zeta)\, d\zeta = -(\nu - 1) \frac{\sin^2 \delta_m}{\delta_m{}^2} \Big[(l_2 - im_2) \frac{d}{dS_2} (\varphi_0{}^\circ + z\, \overline{(\varphi_0{}^\circ)'} + \overline{\psi_0{}^\circ}) +$$
$$+ (l_2 + im_2) \frac{d}{dS_2} (\overline{\varphi_0{}^\circ} + \bar z (\varphi_0{}^\circ)' + \psi_0{}^\circ) \Big] \tag{4.14}$$

It is seen at once from (4.11) and (4.12) that all the $f_{t2}{}^*(s_2) \equiv 0$.

It is seen from an analysis of (4.13) and (4.14) with the boundary conditions (4.4) and (4.9) taken into account, that the right sides of the system (4.10) vanish, and then all the $a_{k2}{}^*(s_2) \equiv 0$.

After analogous computations, we obtain the following boundary conditions for $\varphi_3{}^*$, $\psi_3{}^*$, $a_{k3}{}^*$, $f_{p3}{}^*$:

$$\frac{d}{dS_2} [\varphi_3{}^* + z\, \overline{(\varphi_3{}^*)'} + \overline{\psi_3{}^*}] = - \frac{d}{dS_2} [\varphi_2{}^\circ + z\, \overline{(\varphi_2{}^\circ)'} + \overline{\psi_2{}^\circ}] \tag{4.15}$$

$$\{a_{m3}\} = \frac{a}{2\mu} \delta_m \Big(-\frac{1}{3} I_{m1} + I_{m4} \Big) \Big[(m_2 + il_2) \frac{d}{dS_2} \overline{(\varphi_0{}^\circ + \varphi_0{}^*)''} +$$
$$+ (m_2 - il_2) \frac{d}{dS_2} (\varphi_0{}^\circ + \varphi_0{}^*)'' \Big] \tag{4.16}$$

$$\frac{2\mu}{a} \nu \rho_t{}^3 f_{t3}{}^*(s_2) = a^2 \frac{\nu - 1}{3\nu - 1} \rho_t I_{t10} \Big[(l_2 - im_2) \frac{d}{dS_2} \overline{(\varphi_0{}^\circ + \varphi_0{}^*)''} +$$
$$+ (l_2 + im_2) \frac{d}{dS_2} (\varphi_0{}^\circ + \varphi_0{}^*)'' \Big] \tag{4.17}$$

Here I_{m1}, I_{m4}, I_{t10} are certain constants dependent on the numbers m and t; their values are presented in [1]. They are inessential now.

Therefore $f_{t3}{}^*(s_2)$ and $a_{m3}{}^*(s_2)$ are already nonzero. The subsequent functions $f_{ti}{}^*(s_2)$ and $a_{mi}{}^*(s_2)$ will also be nonzero for $i > 3$.

5. Let us now write down the stresses on the contour Γ_2 of the free hole. They consist of the penetrating biharmonic and reflected states of stress

$$\sigma_{n_2} = \sigma_{n_2}^\circ + \sigma_{n_2}{}^* = -\frac{1}{2} \Big\{ (m_2 + il_2) \frac{d}{dS_2} [\varphi^\circ + \varphi^* + z\, \overline{(\varphi^\circ + \varphi^*)'} + \overline{(\psi^\circ + \psi^*)}] +$$
$$+ (m_2 - il_2) \frac{d}{dS_2} [\overline{(\varphi^\circ + \varphi^*)} + \bar z (\varphi^\circ + \varphi^*)' + \psi^\circ + \psi^*] \Big\} - \lambda^2 a^2 \frac{\nu - 1}{3\nu - 1} \Big(\frac{1}{3} - \zeta^2 \Big) \times$$
$$\times \Big[(m_2 + il_2) \frac{d}{dS_2} \overline{(\varphi^\circ + \varphi^*)''} + (m_2 - il_2) \frac{d}{dS_2} (\varphi^\circ + \varphi^*)'' \Big] +$$
$$+ \frac{2\mu}{a} \lambda^2 \sum_{k=1}^{\infty} [(\nu - 1) p_k(\zeta) + \delta_k{}^2 \alpha_k(\zeta)]\, a_{k3}{}^*(s_2) + \cdots = 0 \tag{5.1}$$

$$\tau_{n_2s_2} = \tau_{n_2s_2}^{\circ} + \tau_{n_2s_2}^{*} = -\frac{1}{2}\Big\{(l_2 - im_2)\frac{d}{dS_2}\,[\varphi^{\circ} + \varphi^{*} + z\,\overline{(\varphi^{\circ} + \varphi^{*})'} + \overline{(\psi^{\circ} + \psi^{*})}] +$$
$$+ (l_2 + im_2)\frac{d}{dS_2}\,[\overline{(\varphi^{\circ} + \varphi^{*})} + \bar{z}\,(\varphi^{\circ} + \varphi^{*})' + \psi^{\circ} + \psi^{*}]\Big\} -$$
$$- \lambda^2 a^2 \frac{\nu - 1}{3\nu - 1}\left(\frac{1}{3} - \zeta^2\right)\left[(l_2 - im_2)\frac{d}{dS_2}\,\overline{(\varphi^{\circ} + \varphi^{*})''} + (l_2 + im_2)\frac{d}{dS_2}\,(\varphi^{\circ} + \varphi^{*})''\right] -$$
$$- \frac{2\mu}{a}\,\nu\lambda^2 \sum_{p=1}^{\infty} \rho_p^2 \cos(\rho_p\zeta)\, f_{p3}^{*}(s_2) + \cdots = 0 \qquad (5.2)$$

$$\tau_{n_2z} = \tau_{n_2z}^{\circ} + \tau_{n_2z}^{*} = -\frac{2\mu}{a}\,\nu\lambda^2 \sum_{k=1}^{\infty} \delta_k \gamma_k(\zeta)\, a_{k3}^{*}(s_2) + \cdots = 0 \qquad (5.3)$$

$$\sigma_{s_2} = \sigma_{s_2}^{\circ} + \sigma_{s_2}^{*} = 2\,[(\varphi^{\circ} + \varphi^{*})' + \overline{(\varphi^{\circ} + \varphi^{*})'}] +$$
$$+ \frac{1}{2}\Big\{(m_2 + il_2)\frac{d}{dS_2}\,[\varphi^{\circ} + \varphi^{*} + z\,\overline{(\varphi^{\circ} + \varphi^{*})'} + \overline{(\psi^{\circ} + \psi^{*})}] +$$
$$+ (m_2 - il_2)\frac{d}{dS_2}\,[\overline{(\varphi^{\circ} + \varphi^{*})} + \bar{z}\,(\varphi^{\circ} + \varphi^{*})' + \psi^{\circ} + \psi^{*}]\Big\} + \lambda^2 a^2 \frac{\nu - 1}{3\nu - 1}\left(\frac{1}{3} - \zeta^2\right) \times$$
$$\times \left[(m_2 + il_2)\frac{d}{dS_2}\,\overline{(\varphi_0 + \varphi^{*})''} + (m_2 - il_2)\frac{d}{dS_2}\,(\varphi^{\circ} + \varphi^{*})''\right] +$$
$$+ \frac{2\mu}{a}\,\lambda^2(\nu - 1)\sum_{k=1}^{\infty} p_k(\zeta)\, a_{k3}^{*}(s_2) + \cdots \qquad (5.4)$$

$$\tau_{s_2z} = \tau_{s_2z}^{\circ} + \tau_{s_2z}^{*} = -\frac{2\mu}{a}\,\nu\lambda^2 \sum_{p=1}^{\infty} \rho_p^2 \sin(\rho_p\zeta)\, f_{p3}^{*}(s_2) + \cdots. \qquad (5.5)$$

$$\sigma_z = \sigma_z^{\circ} + \sigma_z^{*} = \frac{2\mu}{a}\,\nu\lambda^2 \sum_{k=1}^{\infty} q_k(\zeta)\, a_{k3}^{*}(s_2) + \cdots \qquad (5.6)$$

The relationship (5.4) shows that the stress σ_{s_2} at points on the unloaded surface of a hole Γ_2 has the form

$$\sigma_{s_2} = \sigma_{s_20} + \lambda\sigma_{s_21} + \lambda^2\sigma_{s_22} + \cdots \qquad (5.7)$$

hence, the member σ_{s_20} is given by the solution of the corresponding plane problem.

The first order infinitesimal term in λ or σ_{s_21} is also determined by analytic functions $\varphi_1^{\circ}, \psi_1^{\circ}$ and $\varphi_1^{*}, \psi_1^{*}$, which do not however agree with the Kolosov-Muskhelishvili functions since their boundary conditions depend on the function $a_{k1}^{\circ}(s_1)$. Only starting with second order terms in λ will the functions $a_{k3}^{*}(s_2)$ and $f_{p3}^{*}(s_2)$, describing the additional state of stress on the inner contour, enter in the solution.

The stress τ_{s_2z} will be a second order quantity in λ as compared with the main stress characterizing the concentration coefficient σ_{s_20}. The stress $\tau_{n_2s_2}$ in the exact solution is zero on the free hole contour, it is also zero in the plane solution by virtue of condition (4.4). The stress σ_z is zero in the plane theory; in fact it will be a quantity on the order of λ^2 as compared with unity.

Therefore, no matter what characteristics are utilized to calculate the concentration coefficient at a free hole, sufficiently remote from the outer contour, the error in plane theory in the case of slab compression will be at least of first order magnitude in λ as compared with the principal value.

This conclusion differs somewhat from the results obtained in the slab bending case [2]; the error of applied theory there is on the order of λ as compared with unity only in the stress σ_{s_2}. The applied theory introduces a discrepancy of the same order as the quantity being considered in calculating the stress $\tau_{s_2 z}$ in the slab bending case.

BIBLIOGRAPHY

1. Vorovich, I. I. and Malkina, O. S., State of stress in a thick plate. PMM Vol. 31, №2, 1967.
2. Aksentian, O. K. and Vorovich, I. I., On the determination of the stress concentration on the basis of the applied theory. PMM Vol. 28, №3, 1964.
3. Savin, G. N., Stress Concentration around Holes. Moscow-Leningrad, Gostekhizdat, 1951.
4. Vorovich, I. I. and Malkina, O. S., On the accuracy of asymptotic expansions of solutions of elasticity theory problems for a thick slab. Inzh. Zh. MTT №5, 1967.
5. Muskhelishvili, N. I., Some Fundamental Problems of Mathematical Elasticity Theory. 5th ed., Moscow, "Nauka", 1966.
6. Lur'e, A. I., Three-dimensional Problems of Elasticity Theory. Moscow, Gostekhizdat, 1955.

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REPRESENTATION OF SOLUTIONS OF THE GREEN TYPE FOR EQUATIONS OF SHELLS BY THE SMALL PARAMETER METHOD

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A method is presented for asymptotic integration of equations in the theory of shells (convex shells are examined) for the case where the free terms in the equations consist of a Dirac delta function or its derivatives. These solutions, which are solutions represented by a function of the Green type, correspond to the action of concentrated forces or moments on the shell.

At first the analysis is carried out for one equation and then it is shown how the obtained results are extended to the system.

1. Let us examine the linear differential equation containing the small parameter ε which appears in the theory of shells as the relative thickness

$$\varepsilon^s M(w) + L(w) = \delta \tag{1.1}$$

Here M and L are elliptic differential operators with variable coefficients and highest derivatives of orders $2m$ and $2l$, $m > l$, respectively. Without any loss we can write $s = 2(m - l)$.

In the theory of shells the order of operators M and L are equal to $2m = 8$ and $2l = 4$; however, all arguments will be carried out for arbitrary m and l. Since the dimension of the space n does not have any significance with respect to the presented arguments, we shall carry them out for any arbitrary even n (the case of uneven n is examined in an analogous manner).